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Autonomous Broadband Current Meter on a High-Voltage Electrode

N. A. Bogatov

A. V. Gaponov-Grekhov Institute of Applied Physics of the Russian Academy of Sciences
46, Ul'yanov Street, Nizhny Novgorod, 603950, Russian Federation
bogatov@appl.sci-nnov.ru

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Abstract: *The design of an autonomous noise-proof current meter on a high-voltage electrode is proposed, based on the use of an annular non-inductive shunt with its axially symmetrical connection to the input of the data acquisition module. The parameters of two design options are calculated: high-frequency, with a frequency band of 0–10 GHz, but with a lower limit of the measured current, and relatively low-frequency, with a band of 0–100 MHz, and a high limit of the measured current.*

Keywords: *current meter, tubular shunt, cutoff frequency.*

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1. Introduction

In studies of long sparks, as a rule, there is a need to measure the current at a high-voltage electrode. For this purpose, autonomous current meters are used, which are installed on a high-voltage electrode, and the meter is connected to a computer via a fiber optic line [1–4] or via a Wi-Fi link [5–7]. A long spark discharge consists of a set of processes with very different characteristic times. The characteristic time of branching and interaction of streamers resides in the subnanosecond range, and the total time of the discharge process can reach tens of milliseconds. Therefore, shunts are used as a sensitive element in the spark discharge current meters which have 0 Hz as a lower limit of the frequency range. To measure currents up to 100 A, when the shunt resistance can be of the order of 1 Ω or more, low-inductance resistances [1–4] or a set of such resistances connected in parallel in an axially symmetrical design [4] are used. The upper limit of the frequency range of the current meters in [1–4] was 20–70 MHz.

When measuring currents in the kiloampere range, non-inductive tubular shunts with a resistance of the order of units of mΩ are used [5–7]. The upper limit of the tubular shunt frequency range is limited by three factors: skin-effect in the shunt, shunt size, and the inductance of the wires connecting the shunt to the coax connected to input of the data acquisition module. In addition, with a low resistance of the shunt, the quality factor of the oscillatory circuit formed by the inductance of the shunt in the discharge current circuit (the shunt is non-inductive only for the measuring circuit) and the capacitance between the meter body and the electrode, which is parallel to the shunt, substantially increases which leads to parasitic current fluctuations in the shunt and distortion measurements. The upper frequency of the current meter in [5–7], limited by skin-effect, is 27 MHz.

In this paper, we propose a shunt design for a current meter on a high-voltage electrode, which makes it possible to largely overcome the above disadvantages of the shunt used in [5–7] and raise the upper limit of the shunt frequency range to 10 GHz.

2. Shunt design

The main idea of the proposed shunt design is to ensure complete axial symmetry of both the shunt itself and its connection to the input of data acquisition module. The ideal implementation of this idea is the construction shown schematically in Fig. 1a (hereinafter, we will call it 'design **a**'). The shunt has the shape of a ring, the inner and outer diameters of which coincide with the diameters of the inner and outer conductors, respectively, of a coaxial 50-ohm line connecting the shunt to the 50-ohm input of the data acquisition module. The upper frequency band limit of the current meter of such a design can be made equal to the cutoff frequency of the coax (with the appropriate bandwidth of the data acquisition module). However, the small size of the shunt in the absence of a heat sink can severely limit the amplitude and duration of the measured current due to heating of the shunt and changes in its resistance. The greatest heating of the annular shunt occurs at its inner radius. In order to be able to use shunts with a large inner radius, the design shown in Fig. 1b ('design **b**') is proposed. An increase in the size of the shunt inevitably leads to a decrease in the upper frequency.

The following are estimates of the achievable current meter parameters for both of the above shunt designs.

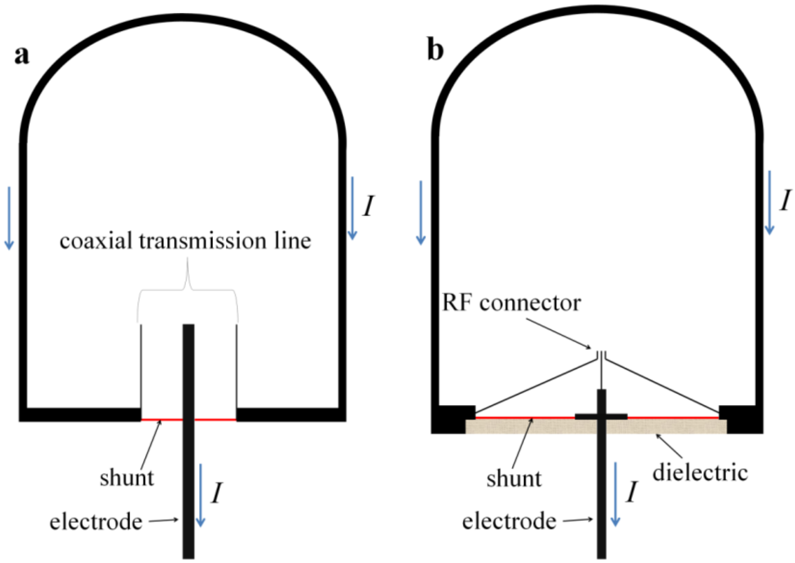

Fig. 1. Two designs of the current meter:
with an annular shunt inside the coaxial transmission line (**a**)
and with a large annular shunt axially symmetrically connected to the coaxial line (**b**)

3. Calculation of ring shunt parameters

Let D and d be the large and small diameters of the shunt, respectively, h is the thickness of the shunt, σ is the conductivity of the shunt material. The resistance of a ring with a diameter r and a width dr is:

$$dR = \frac{dr}{2\pi r h \sigma} \tag{1}$$

The shunt resistance is:

$$R = \frac{1}{2\pi h \sigma} \ln \frac{D}{d} \tag{2}$$

The current density $j(r)$ in the shunt is:

$$j(r) = \frac{I}{2\pi r h} \tag{3}$$

where I is the total current flowing through the shunt (current being measured). The current density j_m in the shunt near its small diameter is:

$$j_m = \frac{I}{\pi d h} \tag{4}$$

The rate of local temperature increase in the absence of thermal conductivity:

$$\frac{dT}{dt} = \frac{j^2}{\sigma C} \tag{5}$$

where C is the heat capacity per unit volume of shunt material. The local heating of the shunt in the absence of heat conduction is equal to:

$$T(r) = T_0 + \frac{1}{\sigma C}\int j(r)^2 dt \tag{6}$$

where T_0 is the shunt start temperature. Integration in (6) is carried out over the entire pulse duration of the measured current. The resistance of the ring dr after the heating is:

$$dR' = \left[1 + \frac{\alpha}{\sigma C(2\pi r h)^2}\int I^2 dt\right]\frac{dr}{2\pi r h\sigma} \tag{7}$$

where α is the temperature coefficient of resistance for the shunt material. The shunt resistance after the heating is:

$$R' = \frac{1}{2\pi h\sigma}\ln\frac{D}{d} + \frac{\alpha}{16\sigma^2 C(\pi h)^3}\left(\frac{1}{d^2} - \frac{1}{D^2}\right) \tag{8}$$

The relative change in the shunt resistance due to heating is:

$$\frac{R'-R}{R} = \frac{\alpha}{8\pi^2\sigma C h^2 \ln\frac{D}{d}}\left(\frac{1}{d^2} - \frac{1}{D^2}\right)\int I^2 dt \tag{9}$$

In the presence of a heat sink in the form of a thick dielectric plate, to which an annular shunt is pressed, the temperature of the shunt will be determined by the balance between heat generation in the shunt and heat flow from the shunt. The characteristic time of temperature equalization in the shunt is $\tau_T = \frac{\Lambda^2}{\varkappa_s}$, where $\varkappa_s$ is the thermal diffusivity of the shunt material, Λ is the characteristic size of the shunt which is $\Lambda = \frac{h}{\pi}$ for a flat layer of thickness h:

$$\tau_T = \frac{h^2}{\pi^2\varkappa_s} \tag{10}$$

If the characteristic time of the current impulse I is much greater than τ_T, then the temperature of the shunt will differ little from the temperature of the surface of the dielectric plate to which the shunt is pressed, and the heating of the shunt will occur together with the heating of the dielectric plate. In this case, the shunt temperature will be approximately equal to the temperature of the substrate surface, which can be found from the solution of the heat conduction equation [9]:

$$T_s(r,t) - T_0 = \frac{1}{2\sqrt{\pi\varkappa_p}}\int_0^t \frac{w(r,\tau)}{\sqrt{t-\tau}}d\tau \tag{11}$$

where $\varkappa_p$ is the thermal diffusivity of the substrate material, $w(r,t) = \frac{j(r,t)^2 h}{\sigma C}$ is the heat flux to the substrate surface. Substituting the current density (3) into (11), one obtain:

$$T_s(r,t) - T_0 = \frac{1}{8\pi^2\sqrt{\pi\varkappa_p}\sigma C h r^2}\int_0^t \frac{I(\tau)^2}{\sqrt{t-\tau}}d\tau \quad (12)$$

Then the relative change in the resistance of the shunt will be equal to:

$$\frac{R'-R}{R} = \frac{\alpha}{16\pi^2\sigma C h \ln\left(\frac{D}{d}\right)\sqrt{\pi\varkappa_p}}\left(\frac{1}{d^2} - \frac{1}{D^2}\right)\int_0^t \frac{I(\tau)^2}{\sqrt{t-\tau}}d\tau \quad (13)$$

The integral in (13) increases during the current flow and decreases after it ends. If the current pulse is rectangular with amplitude I_0 and duration τ_I, then formulas (9) and (13) take the form:

$$\frac{R'-R}{R} = \frac{\alpha}{8\pi^2\sigma C h^2 \ln\left(\frac{D}{d}\right)}\left(\frac{1}{d^2} - \frac{1}{D^2}\right) I_0^2 \tau_I \quad (14)$$

$$\frac{R'-R}{R} = \frac{\alpha}{8\pi^2\sigma C h^2 \ln\left(\frac{D}{d}\right)}\left(\frac{1}{d^2} - \frac{1}{D^2}\right) I_0^2 \tau_I \sqrt{\frac{h^2}{\pi\varkappa_p \tau_I}} \quad (15)$$

As can be seen from formulas (14) and (15), the relative change in the resistance of a shunt with a heat sink will be less than without a heat sink if the current pulse duration τ_I is longer than the characteristic time $\frac{h^2}{\pi\varkappa_p}$ of heating the substrate to a depth equal to the shunt thickness h.

The rise time τ_s and the bandwidth f_s, due to the skin effect in the shunt, are [8]:

$$\tau_s = 0.237\mu\sigma h^2 \qquad f_s = \frac{1.46}{\mu\sigma h^2} \quad (16)$$

where μ is the permeability of the shunt material. To increase the frequency f_s the permeability of the shunt material must be chosen to be minimal, namely equal to the vacuum permeability μ_0, then:

$$f_s = \frac{1.46}{\mu_0\sigma h^2} \quad (17)$$

The cutoff frequency f_c of a coaxial line with the diameters of the outer and inner conductors D and d, respectively, is equal to

$$f_c = \frac{2c}{\pi(D+d)\sqrt{\varepsilon}} \quad (18)$$

where c is the speed of light, ε is the relative permittivity of coax insulator. Formula (18) is directly applicable to the design **a** (Fig.1a), in which the dimensions of the shunt are equal to the dimensions of the coax. In the design **b** (Fig. 1b), the coax is smaller than the shunt, so the cutoff frequency of the coaxial connection of the transmission line to the shunt will again be expressed by formula (18), in which D and d are the outer and inner diameters of the annular shunt. In the latter case, ε = 1. Let us further estimate the parameters of the shunts of both designs shown in Fig. 1.

3.1. Design a

Set the upper bound of the shunt frequency band f_b = 10 GHz. The frequencies f_c and f_s must be no less than f_b, let's make them equal to f_b:

$$\frac{2c}{\pi(D+d)\sqrt{\varepsilon}} = f_b \tag{19}$$

$$\frac{1.46}{\mu_0 \sigma h^2} = f_b \tag{20}$$

For a 50 Ω coax, the D/d ratio is given by:

$$\ln\frac{D}{d} = \frac{5}{6\sqrt{\varepsilon}} \tag{21}$$

With a typical value of ε ≈ 2.3 for coaxial cables, the maximum shunt dimensions determined by equations (19) and (21) are D = 8 mm, d = 4.6 mm. We choose copper as the shunt material (σ = $6\cdot10^7$ Ohm^{-1}m^{-1}). The shunt thickness we find from equation (20): h = 1.4 μm. The shunt resistance at this thickness will be equal to R ≈ 1 mΩ (formula (2)). The temperature coefficient of resistance for copper is $\alpha \approx 4.3\cdot10^{-3}$, and the volumetric heat capacity is C = $3.5\cdot10^6$ J/m^3·K, so formula (9) for the relative change in resistance due to heating of the shunt without a heat sink takes the form:

$$\frac{R'-R}{R} = 7.6\cdot10^{-3}\int I^2 dt \tag{22}$$

If we limit the permissible relative change in the resistance of the shunt by δ, then we obtain the following restriction on the integral of the square of the measured current:

$$\int I^2 dt \leq 130\delta \text{ A}^2\text{·s} \tag{23}$$

In order to ensure heat removal from the shunt, it must be placed on a dielectric plate with high thermal conductivity. An ideal material for this purpose is diamond with its record high thermal conductivity of 2300 W/m·K. Beryllium oxide ceramics has the highest thermal conductivity among industrially produced ceramics, its thermal conductivity is about 200 W/m·K (thermal diffusivity $\varkappa_s \approx$

$5 \cdot 10^{-5}$ m^{2}s^{-1}). The thermal diffusivity of copper is $\varkappa_s \approx 1.15 \cdot 10^{-4}$ m^{2}s^{-1}. Therefore, we use a plate made of this ceramic as a heat sink for our shunt. The characteristic time of a copper layer 1.4 μm thick cooling to the substrate temperature is $\tau_T \approx 2$ ns (formula (10)). The duration of a current close to its maximum value must be much longer than τ_T in order for the heat released in the shunt to be transferred to the substrate. This condition is satisfied with a margin in laboratory long sparks, where the characteristic time of the main current pulse during the return-stroke phase is about 1 μs. In this case, the shunt temperature will be approximately equal to the temperature of the substrate surface, given by formula (12), and the relative increase in shunt resistance will be expressed by formula (13):

$$\frac{R'-R}{R} = 4.3 \cdot 10^{-7} \int_0^t \frac{I(\tau)^2}{\sqrt{t-\tau}} d\tau \quad (24)$$

The condition $\frac{R'-R}{R} \leq \delta$ imposes a limitation on the amplitude and duration of the current:

$$\int_0^t \frac{I(\tau)^2}{\sqrt{t-\tau}} d\tau \leq 2.3 \cdot 10^6 \delta \text{ A}^2\text{s}^{1/2} \quad (25)$$

For a rectangular current pulse with the amplitude I_0 and duration τ_I, condition (25) takes the form:

$$I_0^2 \sqrt{\tau_I} \leq 1.2 \cdot 10^6 \delta \quad \text{A}^2\text{s}^{1/2} \quad (26)$$

For example, with a current pulse duration of 1 μs and measurement error $\delta \leq 0.1$, condition (26) implies a limitation on the measured current $I \leq 11$ kA. The current of laboratory long sparks rarely exceeds this value. The high frequency components of the current have a much smaller amplitude than the main current pulse and do not make a significant contribution to the heating of the shunt.

3.2. Design b

The implementation of design **a** may be impossible due to the lack of a data acquisition module with a bandwidth of up to 10 GHz or the complexity of manufacturing a shunt on a plate of beryllium oxide ceramics. In this case, you can turn to design **b**, which is less high-frequency, but does not require a heat sink for the shunt and makes it possible to measure higher currents.

Let's calculate the parameters of the shunt in design **b**, setting the upper limiting frequency f_b = 100 MHz. It is advisable to choose the shunt material from alloys with high resistivity and low temperature coefficient of resistance.

Constantan has the lowest temperature coefficient among them ($\sigma \approx 2 \cdot 10^6$ Ohm^{-1}m^{-1}, $\alpha \approx 10^{-5}$ K^{-1}, $C \approx 3.64 \cdot 10^6$ J·m^{-3}·K^{-1}), so we will choose it for the manufacture of an annular shunt. The maximum shunt thickness that satisfies condition (17) is equal to $h_{max} = \sqrt{\frac{1.46}{\mu_0 \sigma f_b}}$ = 76 μm. We choose the thickness h = 50 μm, for which $f_s = \frac{1.46}{\mu_0 \sigma h^2} \approx$ 230 MHz. Let's take the following dimensions of the shunt D = 100 mm, d = 50 mm. These dimensions satisfy with a margin the condition (18) $\frac{2c}{\pi(D+d)} > f_b$. The resistance of such a shunt will be equal to R = 1.1 mΩ (formula (2)). If it is required that the relative increase in the resistance of the shunt due to heating does not exceed δ, then from formula (9) the following restriction on the integral of the square of the measured current follows:

$$\int I^2 dt \leq 5 \cdot 10^7 \delta \ \text{A}^2\cdot\text{s} \quad (27)$$

More stringent than (27) is the restriction on the maximum temperature of the shunt 1100 °C near its inner diameter, where the current density is maximum:

$$T\left(\frac{d}{2}\right) - T_0 = \frac{1}{\sigma C (\pi d h)^2} \int I^2 dt \leq \ 800\ \text{K} \qquad \int I^2 dt \leq 3.5 \cdot 10^5\ \text{A}^2\cdot\text{s} \quad (28)$$

With a current duration of 1 μs, the current amplitude measured by such a shunt with an error of less than 1% can reach 600 kA. The shunt voltage at this current will be about 700 V, so it must be fed to the data acquisition module input through a voltage divider. To expand the dynamic range of the meter, it is advisable to use two or more registration channels with different sensitivities connected in parallel to the shunt. Thus, it is possible to realize the range of measured currents of the meter from 1 A to 600 kA.

4. Discussion

Specific shunt designs, for which the calculation of parameters was performed in this article, are designed to measure current in laboratory long sparks, in which currents can reach tens of kiloamperes. However, the shunt of the design **a** can also be successfully used to study low-current discharges, if it is necessary to measure the current in a wide frequency range. To measure weak currents, the shunt resistance must be increased by reducing the thickness of the shunt h. The substrate of such a thin ring shunt in this case can be made of any high-frequency dielectric, not necessarily high thermal conductivity, since there is no need for a heat sink for the shunt.

The upper frequency of 10 GHz, which we chose to calculate the parameters of the design **a**, for a ring shunt is not the maximum possible. By reducing the size of the coaxial line and the corresponding ring shunt, it is possible to increase the cutoff frequency even further. In this case, the limitation on the amplitude and duration of the measured current (conditions (23) and (25)) will be tightened.

5. Conclusion

The proposed design **a** of a pulsed current meter based on a ring shunt on a high-voltage electrode makes it possible to measure the current in a very wide frequency range (0–10 GHz). The amplitude and duration of the measured current is limited by condition (25). For example, with a current pulse duration of 1 μs, the current amplitude should not exceed 11 kA so that the measurement error does not exceed 10%.

The current meter of design **b** allows you to measure the pulsed current with the maximum value of the integral of the squared current over time $3.5 \cdot 10^5 \delta$ $A^2 \cdot s$ in the frequency range (0–200 MHz). With an acceptable measurement error of 1% ($\delta \leq 0.01$) and a current pulse duration of 1 μs, the amplitude of the measured current can reach 600 kA.

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Information about the author

Nikolai A. Bogatov, scientific reseacher, A. V. Gaponov-Grekhov Institute of Applied Physics of the Russian Academy of Sciences, Nizhny Novgorod, Russian Federation, ORCID 0000-0001-5034-888X.