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Surface Plasmons and Diffraction of Plane Waves in Metallic Films and Graphene Structures

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Abstract: *We consider the electric and magnetic surface plasmons in the structures with dielectric and metallic layers, or graphene sheets. The conditions of existence of backward and forward waves and the slow and fast plasmon-polaritons are obtained. We also consider the plasmons and plane wave diffraction for the layer of hyperbolic metamaterial.*

Keywords: *diffraction, metallic film, graphene, plasmons, hyperbolic metamaterial.*

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Поверхностные плазмоны и дифракция плоских волн в металлических пленках и графеновых структурах

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Аннотация: *Рассмотрены электрические и магнитные поверхностные плазмоны в структурах с диэлектрическими и металлическими слоями или листами графена. Получены условия существования обратной и прямой волн, медленных и быстрых плазмон-поляритонов. Рассмотрены также плазмоны и дифракция плоских волн для слоя гиперболического метаматериала.*

Ключевые слова: *дифракция, металлическая пленка, графен, плазмоны, гиперболический метаматериал.*

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1. Introduction

The slow surface plasmons in metallic films and along metallic surfaces are usually considered without dissipation, so the dielectric constant of metal is real and negative. In this paper we consider the symmetric lossy plane-layered structures with metallic and dielectric layers and two-dimensional conductive films or two-dimensional electron gas with tensor conductivity. We use as example the graphene conductivity tensor, and for metallic layers we use the Drude-Lorentz model. We also introduce the surface conductivity for thin metallic and dielectric films. The goals of this paper are to get the conditions for existence of forward and backward waves, to get the conditions for existence of fast and low surface plasmons (SPs) and to determine the connection of dispersion equation (DE) or SP existence with the plane wave diffraction without reflection.

2. Problem Statement

The forward and backward plasmons are possible in symmetrical structures without dissipation [1–3]. As in [1–3] the dissipation is not considered, here we show, that there are also in dissipative structures. If the phase and energy movement directions are matched, the plasmon is forward, but if the other case – backward. We take dielectric permittivity (DP) of metal in the form $\varepsilon(\omega)=\varepsilon_L-\omega_p^2/(\omega^2-i\omega\omega_c)$, $\varepsilon(\omega)=\varepsilon_L-\omega_p^2/(\omega^2-i\omega\omega_c)$, where interband transitions and the polarization of the lattice are taken into account in ε_L. We will continue to assume this value is real, characteristically up to optical frequencies. Then the metal model without dissipation corresponds to the zero frequency of collisions $\omega_c=0$. The backward surface plasmon (SP) in this case corresponds to the anomalous negative dispersion. In the case $\omega_c\neq 0$ it is impossible to classify backward surface plasmon (BSP) by anomalous negative dispersion or negative group velocity (GV). In this case, the SP character (backward or forward) is determined by the direction of the Pointing vector relative to the movement of phase. We consider the DP ε_d as real, i.e. we not take into account the dispersion and therefore dissipation. There are the ratios: $k_x^2+k_y^2+k_z^2=k_0^2\varepsilon(\omega)$ in metal and $k_x^2+k_y^2+\tilde{k}_z^2=k_0^2\varepsilon_d$ in dielectric for the finite along the *y*-axis structures. If there is vacuum instead dielectric, that $\varepsilon_d=1$, and we denote. We also consider the symmetric on *y* functions based on the type $\cos(k_y y)$ and use the decompositions

$$\begin{pmatrix} E_x(x,y,z) \\ H_x(x,y,z) \end{pmatrix} = A_{[e,m]}^{(e,h)} \exp(-ik_x x)\cos(k_y y)\begin{bmatrix} \sin(k_z z) \\ \cos(k_z z) \end{bmatrix}. \tag{1}$$

The upper index "*e*" corresponds to E-SP (TM-waves), for which $H_x=0$. The Superscript "*h*" corresponds to H-SP (TE-waves), for which $H_x=0$. Subscript "*e*" corresponds to an electric wall at the center $E_x=0$, i.e., the taking of the sine in (1). Subscript "*m*" corresponds to the magnetic wall, i.e. taking the cosine. For example, the amplitude A_m^e corresponds to even function E_x of *z*. Further, we use the value $k_y=\pi/w$. In the case $w\to\infty$ the relations are accurate, and the structure of the plane-layered. With a large but finite width we get some approximation working better than more w/t. Strictly, these structures can be analyzed by the method of integral equations defined on the cross section, using the metal decomposition of type (1) for functions with different $k_{yn}=(2n-1)\pi/w$ and $k_{zn}=\sqrt{k_0^2\varepsilon-k_x^2-k_{yn}^2}$. For completeness we should add the functions with $k_{zn}=(2n-1)\pi/t$ and $k_{yn}=\sqrt{k_0^2\varepsilon-k_x^2-k_{zn}^2}$. We consider that $k_x=k_x'-ik_x''$. In dissipative structures the SP forward if $k_x'k_x''>0$, and backward *t*, if $k_x'k_x''<0$.We introduce normalized to the impedance of the vacuum $Z_0=\sqrt{\mu_0/\varepsilon_0}$ wave impedance $\rho^e=k_z/(k_0\varepsilon)$ for the E-SP and $\rho^h=k_0/k_z$ for H-SP in the metal. For dielectric we do the substitution $\varepsilon\to\varepsilon_d$ and denote impedances $\tilde{\rho}^e$, $\tilde{\rho}^h$. For vacuum we do the substitution $\varepsilon\to 1$ and denote ρ_0^e, ρ_0^h. The propagation constants also denoted by the symbols *e* and *h*. For SP with an electric wall in the center the normalized input impedance at the border of the metal film 1 is $\rho_{in}^{(e,h)}(t/2)=i\rho^{(e,h)}\tan(k_z t/2)$, therefore, the dispersion equations (DEs) are $\rho_0^{(e,h)}=i\rho^{(e,h)}\tan(k_z t/2)$ and correspond to even components (with respect to *z*-components) and odd with respect to one *z*-component. Typically, these SPs are classified as the even with respect to the transverse component E_z, H_y, and are called symmetric [1]. For SPs with magnetic wall in the center of the metal film 1 the normalized input impedances at the border are $\rho_{in}^{(e,h)}(t/2)=-i\rho^{(e,h)}/\tan(k_z t/2)$, and the are DEs $\rho_0^{(e,h)}=-i\rho^{(e,h)}/\tan(k_z t/2)$. This DEs are called antisymmetric. For structure 2 the input impedance $\rho_{in}^{(e,h)}(t/2)$ should be transformed to the thickness of the dielectric:

$$\rho_0^{(e,h)}=\rho_{in}^{(e,h)}(d+t/2)=\tilde{\rho}^{(e,h)}\frac{\rho^{(e,h)}[i\tan(k_z t/2)]^{\pm1}+i\tilde{\rho}^{(e,h)}\tan(\tilde{k}_z d)}{\tilde{\rho}^{(e,h)}+i\rho^{(e,h)}[i\tan(k_z t/2)]^{\pm1}\tan(\tilde{k}_z d)}. \qquad (2)$$

The DEs (2) describe the odd (the sign “+”) and even (“–“) SPs. Des for structure 4 are obtained from (2) by substitutions of the parameters of metal on the dielectric parameters and vice versa. The DEs for hollow waveguide 3 without dielectric are resulted from the conditions $\rho^{(e,h)}=\rho_0^{(e,h)}[i\tan(k_{0z}t/2)]^{\pm1}$. It is very interesting if there are the dielectric films on the wide walls of the waveguide 3. This case corresponds to DE (2) in which one must change the properties of a metal and vacuum. Let us denote the input impedance of all structures on the last partition of boundaries as $\rho_{in}^{(e,h)}$. If boundary is bordered by vacuum, the DE can be written as

$$k_x^{(e,h)}=k_0\sqrt{1-\left[\rho_{in}^{(e,h)}\right]^{\pm2}} \qquad (3)$$

The upper sign in (3) corresponds to E-SP and lower – to H-SP. If the boundary is bordered with metal, then

$$k_x^e = k_0\sqrt{1-\left[\varepsilon(\omega)\rho_{in}^e\right]^2}, \qquad k_x^h = k_0\sqrt{1-\left[\rho_{in}^h\right]^{-2}}. \tag{4}$$

Denoting $\rho_{in}^{(e,h)} = \rho' + i\rho''$, we see from (3) that that in the dissipative structures ($\rho' > 0$) the E-SP is forward if the input impedance is inductive ($\rho'' > 0$), and it is

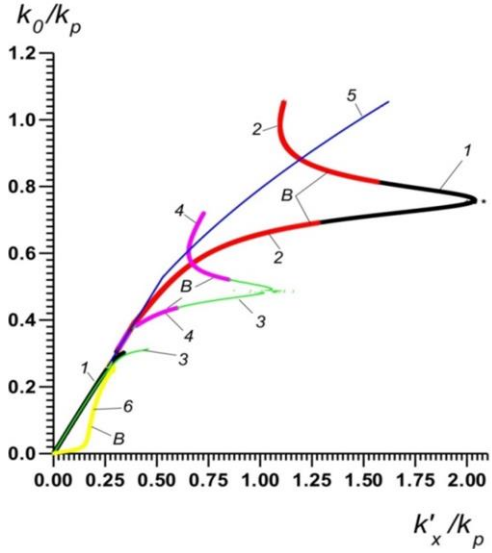

Fig. 1. Forward (curves 1, 3, 5) and backward (symbol B, curves 2, 4, 6) plasmon-polariton branches for electric antisymmetric (1, 2, 3, 4) and symmetric magnetic (5, 6) of the plasmons in the layer of 50 nm (1, 2), 100 nm (3, 4) and 10 nm (5, 6).

Рис. 1. Прямая (кривые 1, 3, 5) и обратная (символ B, кривые 2, 4, 6) плазмон-поляритонные ветви для электрических антисимметричных (1, 2, 3, 4) и симметричных магнитных (5, 6) плазмонов в слое 50 нм (1, 2), 100 нм (3, 4) и 10 нм (5, 6)

backward, if the impedance is capacitive ($\rho'' < 0$). Since the input impedance depends on the square of the wave number (3), the following inequality must be preserved for this type of E-SP. It is convenient to take the positive direction in the direction of the movement phase, i. e. $k_x'^{(e,h)} > 0$. Then for forward E-SP $\rho'' > 0$ at $k_x''^{(e,h)} > 0$, and for backward one $\rho'' < 0$ at $k_x''^{(e,h)} < 0$. For H-SP (3) and the second of the relations (4) we see that H-SP is direct when $\rho'' < 0$ and backward when $\rho'' > 0$, because the impedances in they DEs are replaced by admittances. Indeed, for E-SP we input complex number $z = 1-\rho'^2+\rho''^2-2i\rho'\rho''$, which stands under the sign of the square root. The branch of root specified by condition $\mathrm{Re}\left(\sqrt{z}\right)>0$ yields the above conditions. For the first equation (4) we have

$$z=\left[1-\left(\rho'\varepsilon'+\rho''\varepsilon''\right)^2+\left(\varepsilon'\rho''-\rho'\varepsilon''\right)\right]-$$
$$-2i\left(\rho'\varepsilon'+\rho''\varepsilon''\right)\left(\varepsilon'\rho''-\rho'\varepsilon''\right).$$

Here $\varepsilon'<0$, $\varepsilon''>0$, $\rho'>0$. Obviously, SP is forward, if $\mathrm{Im}(z)<0$, and backward if $\mathrm{Im}(z)>0$. Because $\varepsilon'\rho''-\rho'\varepsilon''<0$, for a backward SP it is required $\rho'\varepsilon'+\rho''\varepsilon''>0$, or $\rho''/\rho'>\left|\varepsilon'\right|/\varepsilon''$. Let consider the simplest case of E-SP along the dielectric waveguide with metal plates. Here $\rho=\rho_{in}^e=\tilde{k}_z^e\left(\varepsilon_d k_0\right)^{-1}\left[i\tan\left(\tilde{k}_z^e t/2\right)\right]^{\pm1}$. We write $\tilde{k}_z^e t/2=i\tilde{\theta}=it\sqrt{\left(k_x^e\right)^2-k_0^2\varepsilon_d}/2$ and consider the SP as slow, i. e. $k_x'^e>k_0\sqrt{\varepsilon_d}$, while the loss is small: $\tilde{\theta}=\tilde{\theta}'-i\tilde{\theta}''$, $\tilde{\theta}''<<\tilde{\theta}'$. Then in the odd case

$$\rho''/\rho'=\frac{\tilde{\theta}'\sin\left(2\tilde{\theta}'\right)-\tilde{\theta}''\sinh\left(2\theta''\right)}{\tilde{\theta}'\sinh\left(2\tilde{\theta}''\right)+\tilde{\theta}''\sin\left(2\tilde{\theta}'\right)}\approx\frac{\tilde{\theta}'\sin\left(2\tilde{\theta}'\right)}{\left(2\tilde{\theta}'+\sin\left(2\tilde{\theta}'\right)\right)\tilde{\theta}''}.$$

This condition is satisfied for the slow SP near the plasmon resonance at the frequency $\omega_s=\omega_p/\sqrt{\varepsilon_L+1}$, for which the ratio $\left|\varepsilon'\right|/\varepsilon''$ is not very large, because at $\omega\approx\omega_s$ we have

$$\frac{\left|\varepsilon'\right|}{\varepsilon''}=\frac{\omega_p\omega_c\left(\varepsilon_L+1\right)^{3/2}}{\omega_p^2+\omega_c^2\left(\varepsilon_L+1\right)}\approx\frac{\omega_c\left(\varepsilon_L+1\right)^{3/2}}{\omega_p}.$$

The typical values are $\varepsilon_L\sim10-20$, $\omega_c/\omega_p\sim10^{-2}-10^{-3}$. In the even case

$$\frac{\rho''}{\rho'}=\frac{\tilde{\theta}'\sin\left(2\tilde{\theta}'\right)+\theta''\sinh\left(2\tilde{\theta}''\right)}{\tilde{\theta}''\sin\left(2\tilde{\theta}'\right)-\tilde{\theta}'\sinh\left(2\tilde{\theta}''\right)}\approx\frac{-\tilde{\theta}'\sin\left(2\tilde{\theta}'\right)}{\left(2\tilde{\theta}'-\sin\left(2\tilde{\theta}'\right)\right)\tilde{\theta}''},$$

therefore, the inequality cannot be.

3. Results

Consider the DE for the metal strip waveguide. We have the DEs:

$$k_{ex}^e=k_0\sqrt{\frac{\varepsilon^2-\varepsilon\tanh^2(\theta)}{\varepsilon^2-\tanh^2(\theta)}},\quad k_{mx}^e=k_0\sqrt{\frac{\varepsilon^2\tanh^2(\theta)-\varepsilon}{\varepsilon^2\tanh^2(\theta)-1}},\tag{5}$$

$$k_{ex}^h=k_0\sqrt{\frac{\varepsilon-\tanh^2(\theta)}{1-\tanh^2(\theta)}},\quad k_{mx}^h=k_0\sqrt{\frac{1-\varepsilon\tanh^2(\theta)}{1-\tanh^2(\theta)}}.\tag{6}$$

Here θ is following from the same formula, but with DP of metal. In the absence of dissipation we have $\varepsilon<0$, and SP k_{mx}^e is infinitely slow at the frequency where $\varepsilon^2\tanh^2(\theta)=1$, i. e. at $\omega=\omega_{sm}<\omega_p/\sqrt{\varepsilon_L+1}$. This frequency decreases with decreasing *t*. In pursuit of this frequency from below always

$\varepsilon^2 > \tanh^2(\theta)$. Above this frequency there is the forbidden band of bandgap. Behind her in the field $0 < \varepsilon < 1$ the polariton fast: $k_{ex}^e \approx k_0\sqrt{\varepsilon}\left[1-\varepsilon(1-\varepsilon)/\left(2\tanh^2(\theta)\right)\right]$, and near the frequency of the plasmon resonance $\omega \approx \omega_s$ we have $k_{ex}^e \approx k_0\sqrt{\varepsilon} = k_0\sqrt{\omega_p^2/\omega^2 - \varepsilon_L}$. SP is weak slow at low frequencies, where $k_{ex}^e < k_{mx}^e$. A strong slowdown beginning to emerge near the frequency $\omega_s = \omega_p/\sqrt{\varepsilon_L + 1}$ where $\tanh^2(\theta) = \varepsilon^2 = 1$. At the approach to it from below the dispersion is normal and performed $\varepsilon^2 > \tanh^2(\theta)$. It's made in the case $\omega > \omega_s$, but the dispersion is anomalous: the retardation decrease with the increase in frequency. Higher it the dispersion becomes anomalous and negative. At the frequency $\omega_p/\sqrt{\varepsilon_L - 1}$, where $\varepsilon = 1$, the polariton becomes fast. If you are not getting comprehensive , ñ becomes leaky, there are radiation losses. At the condition $\omega > \omega_p/\sqrt{\varepsilon_L - 1}$ the term $\tanh^2(\theta)$ is complex, the polariton becomes leaky, and the losses arise. In the point ω_s the dispersion curve is not differentiable. Consideration of losses leads to its differentiability, the slowdown in the neighborhood of frequencies ω_s is finite, and GV goes through infinity and changes the sign. This is also the case for k_{mx}^e lower ω_s, and in the bandgap the propagation occurs for k_{mx}^e with anomalous negative dispersion. Polariton k_{ex}^h may not be slow and surface. SP is very slow k_{mx}^h at low frequencies, when $-\varepsilon\tanh^2(\theta) >> 1$ and $\tanh^2(\theta) << 1$. However, at these frequencies one should take into account the dissipation. SP has forward and backward portions of the dispersion branches. It has no component E_x and does not interact with the longitudinal electron beams.

SP k_{mx}^e for structure 2 can be backward when the dielectric layer transforms the inductive impedance to the capacitive Fig. 1. A suitable analytical condition is quite cumbersome, so here it omitted. Really thin metal film waveguides of the type 1 and 2 should be considered in some dielectric medium with DP $\tilde{\varepsilon}$. To do this, in all relationships you should make the change $k_0 \to k = k_0\sqrt{\tilde{\varepsilon}}$. Then the direct SP moving in the film 1 and greeting on your way the strip of section 2, when the waveguide 1 is overlaid with dielectric plates with a DP ε_d, can get the focusing if he becomes the backward. It should be noted that the initiation of SP by the finite source, e. g. a laser beam through a prism gives divergent SP, which, after such metalens will be converging. It should be noted that for the waveguide 3 the above obtained relations are exact in the approximation of perfectly conducting narrow walls.

4. Conductivity of 2D Structures

In thin metal layers (films) $|z| < d$ there are slow surface plasmons (SP) of the electric type in case of presence in the plane of symmetry $z = 0$ of the magnetic wall, and the slowdown increases with decreasing thickness d. For application the films with a thickness of a few nanometers are interesting. In the case of the plane of symmetry of the magnetic wall electric E-SP slower (lower frequency plasmon resonance ω_s) than E-SP electric wall in the center z=0 of the film [1, 5, 6]. In the approximation of absence of dissipation E-SP with electric wall has two branches, suitable to the frequency below (with normal dispersion) and the top (with the anomalous negative dispersion). The latter case corresponds to a backward wave [1]. However, when we have a dissipation, the branches are closed, and in the vicinity of the plasmon resonance ω_s both from the bottom and top SP are direct, and the loss during the motion along the x-axis high: $k_x''(\omega_s) \approx k_x'(\omega_s)$ [5, 6]. The polarization is always possible to choose so that $k_y = 0$. For forward SP $k_x' k_x'' > 0$, and for the backward one $k_x' k_x'' < 0$, i.e. the energy (attenuation of waves) moving back to phase because $k_x = k_x' - i k_x''$. We denote the film thickness $t = 2d$. A conductive film is two-dimensional, if its thickness is significantly less than the mean free path $(t << \lambda_c)$ and penetration depth of the electromagnetic wave ($t << \delta$). Usually in this case also $t << \lambda$, where λ is the wavelength in vacuum. This structure is called two-dimensional electron gas (DEG) and describes a two-dimensional (surface) conductivity $\sigma = \varsigma t$, where ς is the bulk conductivity of the solid sample, which determines the current density $\mathbf{J} = \varsigma \mathbf{E}$. In view of the above the DEG is described by the surface current density $\mathbf{J}_S$ is determined by the conditions $\mathbf{J}(\mathbf{r}_S, \nu) = \mathbf{J}_S(\mathbf{r}_S)\delta(\nu) = \sigma \mathbf{E}_S(\mathbf{r}_S, 0)\delta(\nu)$, where the index S indicates the projection of the vector on the surface of the film, and ν is the normal coordinate is counted from it. From the point of view of electrodynamics such film has not a finite transverse size, and the volume integral is reduced to surface one. The conditions for DEG depend on the temperature, frequency, purity of the sample, the surface of the film and are usually granted for good conductive metals with thicknesses from a few to tens of nanometers. Usually the DEG films are located between the dielectric layers or on such layer. The use of bulk materials conductivity or permittivity for estimation of DEG parameters is an approximation. A rigorous approach requires the solution of complex quantum tasks including borders and dielectric layers, because the density of states, frequency of collisions, the number of modes of conductivity, defining the conductivity σ, substantially change it at small thicknesses, so that the ratio $\sigma = \varsigma t$ is strictly true for thick films. However, there is a natural

DEG structure, e. g., graphene [7–11], graphenes [12] and some others. For graphene σ is defined in several works [7–11] based on the approach of Kubo-Greenwood and the method of dynamic Green's functions. Tensor dynamic conductivity of graphene when exposed to a plane monochromatic wave is determined by the integral in momentum space [7–11]. His approximate evaluation on the part of the Brillouin zone in the vicinity of two Dirac points in the approximation of linear dispersion $\mathrm{E}(\mathbf{p}) = \pm v_F|\mathbf{p}|$, $\mathbf{p} = \hbar\mathbf{q}$, $v_F \approx c/300$ leads to a simple formula Kubo [5]: $\sigma=\sigma_{intra}+\sigma_{inter}$ where

$$\begin{aligned}&\sigma_{\mathrm{intra}}(\omega,\mu,\omega_c,T)=\\&=-2ie^2k_BT\left[\pi\hbar^2(\omega-i\omega_c)\right]^{-1}\ln\left(\cosh\left(\mu/(2k_BT)\right)\right)\end{aligned}, \tag{7}$$

$$\begin{aligned}&\sigma_{\mathrm{inter}}(\omega,\mu,\omega_c,0)=\\&=-ie^2/(4\pi\hbar)\ln\left(\left[2|\mu|-(\omega-i\omega_c)\hbar\right]/\left[2|\mu|+(\omega-i\omega_c)\hbar\right]\right)\end{aligned}. \tag{8}$$

For (7) and (8) the Fermi-Dirac function $f(\mathrm{E}) = \left[\exp(\mathrm{E}-\mu_c)/k_BT+1\right]^{-1}$ is used in with μ_c is the chemical potential. To take into account the spatial dispersion in [11] under the approximation of Bhatnagar-Gross-Krook (BGK) with account of drift current and the relaxation time approximation (RTA) model for tensor conductivity are given. The BGK model gives the conductivity tensor with account of spatial dispersion. The RTA model gives some differ results.

5. Plane Wave Diffraction

Let consider the located in vacuum conductive DEG film, the center of which is located at $z=0$. Let an electromagnetic wave $\mathbf{E}(t,\mathbf{r}) = \mathbf{E}_0\exp(i\omega t-\mathbf{kr})$ in vacuum, in which $\mathbf{k}^2 = k_0^2$, falls at the considered film with tensor surface conductivity $\hat{\sigma} = \hat{\xi}/Z_0$ where $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 120\pi$ Ω. The magnetic field is determined by the same dependence with amplitude $\mathbf{H}_0 = \mathbf{k}\times\mathbf{E}_0/Z_0$. We solve the problem by two ways. The first is based on electrodynamic Green's function (GF). The second will use the mode matching technique. The wave creates in the film the surface current density $\mathbf{J}_S(x,y)$, which we write using the volumetric current density in the form $\mathbf{J}(x,y)= \mathbf{J}_S(x,y)\delta(x)$. We take the scalar GF [13], denoting $\mathbf{R} = \mathbf{r}-\mathbf{r}'$, $R = |\mathbf{R}| = \sqrt{X^2+Y^2+Z^2}$, $X = (x-x')$, $Y = (y-y')$, $Z = (z-z')$, $k_z = \sqrt{k_0^2 - k_x'^2 - k_y'^2}$.

GF is the decomposition on non-uniform plane-wave, including evanescent ones in the case $k_x'^2 + k_y'^2 > k_0^2$. We have an electric vector-potential, equal to the integral over the surface from $G(\mathbf{r}-\mathbf{r}'_S)\mathbf{J}_S(\mathbf{r}'_S)$, where we have moved from the volume integration to a surface integral over the infinite plane (x,y). In force of local link $\mathbf{J}_S(x,y) = \hat{\sigma}\mathbf{E}_S(x,y)$ the integral is calculated explicitly and has the form:

$\mathbf{A}(\mathbf{r}) = \hat{\sigma}\mathbf{E}_S g$, $g = \exp(-ik_x x - ik_y y - ik_z|z|)/(2ik_z)$. Here $\mathbf{E}_S$ is the amplitude of the total tangential electric field on the surface. Hence, we find the tangential electric $\mathbf{E}_d = (\nabla\nabla\cdot\mathbf{A} + k_0^2\mathbf{A})/(i\omega\varepsilon_0)$ and magnetic fields $\mathbf{H}_d = \nabla\times\mathbf{A}$ of diffraction with amplitudes $E_{(x,y)d}(\mathbf{r}) = \overline{E}_{(x,y)d}\exp(-ik_x x - ik_y y - ik_z|z|)$, were

$$\overline{E}_{xd} = \frac{k_x^2(\sigma_{xx}E_x + \sigma_{xy}E_y) + k_x k_y(\sigma_{yx}E_x + \sigma_{yy}E_y) - k_0^2(\sigma_{xx}E_x + \sigma_{xy}E_y)}{2\omega\varepsilon_0 k_z},$$

$$\overline{E}_{yd} = \frac{k_y^2(\sigma_{yy}E_y + \sigma_{yx}E_x) + k_x k_y(\sigma_{xy}E_y + \sigma_{xx}E_x) - k_0^2(\sigma_{yy}E_y + \sigma_{yx}E_x)}{2\omega\varepsilon_0 k_z}.$$

Further we believe $\sigma_{xy}=\sigma_{yx}$. The amplitude of the full field on the tape is equal to the sum of the respective amplitudes of the incident and diffracted fields: $E_x = E_{0x} + \overline{E}_{xd}$, $E_y = E_{0y} + \overline{E}_{yd}$. These conditions lead to a system of linear algebraic equations (SLAE) with has two unknown amplitudes E_x and E_y: $\hat{A}E = E_0$, $E = (E_x, E_x)$, $E_0 = (E_{0x}, E_{0x})$, allowing to express the full amplitude using the amplitude of the incident field. As a result $E_x = A_{xx}^{-1}E_{0x} + A_{xy}^{-1}E_{0y}$, $E_y = A_{yx}^{-1}E_{0x} + A_{yy}^{-1}E_{0y}$, where we have introduced the matrix elements of the inverse matrix $\hat{A}$:

$$A_{xx} = \left[1 + (k_0^2 - k_x^2)\xi_{xx} - k_x k_y \xi_{xy}\right]/(2k_0 k_z),\ A_{xy} = \left[(k_0^2 - k_x^2)\xi_{yy} - k_x k_y \xi_{xy}\right]/(2k_0 k_z).$$

For the rest of the coefficients matrix we should make the change $x \leftrightarrow y$. When choosing a polarization $E_{0y}=0$, i. e. along the x axis and in the normalization $E_{x0}=1$ we have the reflection coefficient $R = \overline{E}_{xd}$, transmission coefficient $T=1+R$, and the coefficient of transformation of the orthogonal polarization $Q = \overline{E}_{yd}$. The solution to the problem of diffraction takes place if the matrix of SLAE is not a special, i. e. $\det(\hat{A}) \neq 0$. We have the relations $R = -1 + A_{11}^{-1} = -1 + a_{22}/\det(\hat{A})$, $Q = A_{12}^{-1}$, which we can present or in the following forms:

$$R = \frac{k_x^2(\sigma_{xx}A_{xx}^{-1} + \sigma_{xy}A_{yx}^{-1}) + k_x k_y(\sigma_{yx}A_{xx}^{-1} + \sigma_{yy}A_{yx}^{-1}) - k_0^2(\sigma_{xx}A_{xx}^{-1} + \sigma_{xy}A_{yx}^{-1})}{2\omega\varepsilon_0 k_z},$$

$$Q = \frac{k_y^2(\sigma_{yy}A_{yx}^{-1} + \sigma_{yx}A_{xx}^{-1}) + k_x k_y(\sigma_{xy}A_{yx}^{-1} + \sigma_{xx}A_{xx}^{-1}) - k_0^2(\sigma_{yy}A_{yx}^{-1} + \sigma_{yx}A_{xx}^{-1})}{2\omega\varepsilon_0 k_z}.$$

The condition $\det(\hat{A}) = 0$ is the dispersion relation (DR) for surface plasmons along the film. It determines the dispersion $k_0 = f(k_x, k_y)$, or $\omega = \omega(\mathbf{k}_S)$ believing that DR resolved relative to the wave number k_0. Because of the Fresnel equations we have $k_z = \sqrt{k_0^2 - k_x^2 - k_y^2}$. Because the frequency is real, we have the condition $\mathrm{Im}(f(k_x, k_y)) = 0$. Therefore we have two real equations to determine the two complex quantities, for example, k_x and k_y (equation Fresnel allows you to choose any pair components of vector **k**). The goal is to get the dependencies $k_x = k_x(k_0)$, $k_y = k_y(k_0)$. However, in the DR these values relate and depend on directions of phase and energy. Let SP has the complex surface wave vector $\mathbf{k}_S = \mathbf{k}'_S - i\mathbf{k}''_S$, the real part of which (indicated by bar) forms an angle φ with the x axis. The direction of phase movement is determined by the unit vector $\mathbf{n}_{0p} = \mathbf{k}'_S / k'_S$, where $k'_S = |\mathbf{k}'_S| = \sqrt{k'^2_x + k'^2_y}$. Then $k'_x = k'_S \cos(\varphi)$, $k'_y = k'_S \sin(\varphi)$. The unit vector $\mathbf{n}_{0E} = \mathbf{k}''_S / k'_S$ defines the direction of motion of energy that moves in the direction of attenuation of the wave. Here $k''_S = |\mathbf{k}''_S| = \sqrt{k''^2_x + k''^2_y}$. Also we have $\mathbf{n}_{0E} = \tilde{\mathbf{S}} / |\tilde{\mathbf{S}}|$, where $\tilde{\mathbf{S}}$ is the integral of the Poynting vector along the transverse coordinate z. The specified relationship exists not only for the surface plasmon, and (as a limit) for deriving polariton. In the General case $\mathbf{k}_{0E} \neq \mathbf{k}_{0p}$ and $\mathbf{k}_{0E} \neq \mathbf{v}_g / |\mathbf{v}_g|$, where $\mathbf{v}_g = \nabla_{\mathbf{k}'_S} \omega(\mathbf{k}_S)$ is the vector group velocity (GV). Because of dissipation the Leontovich–Lighthill–Rytov theorem [14] is not performed, and the GV does not define the movement of energy. Let the energy direction is determined by the angle ψ relative to the x axis. Then $k''_x = k''_S \cos(\psi)$, $k''_y = k''_S \sin(\psi)$. At the same time $\mathbf{k}_S = \mathbf{x}_0 k_x + \mathbf{y}_0 k_y$, and complex DR is splitting into two real ones: $\mathrm{Re}(\det(\hat{A})) = 0$, $\mathrm{Im}(\det(\hat{A})) = 0$. By setting the direction of the surface plasmon $\mathbf{n}_p = \mathbf{k}'_S / |\mathbf{k}'_S|$, calculating the Poynting vector **S** and adding these two real equations by the third $\mathbf{n}_E = \tilde{\mathbf{S}} / |\tilde{\mathbf{S}}|$, it is possible to define the k'_S, k''_S, and the complex wave vector $\mathbf{k}_S = k'_S \mathbf{n}_p - i k''_S \mathbf{n}_E$. Its real part k'_S is the phase constant along the motion direction, and a negative imaginary part k''_S determines the energy flow and attenuation in the direction $\mathbf{n}_E$. If $\mathbf{n}_p \mathbf{n}_E > 0$, then the direction of movement of phase and energy form an acute angle, and the plasmon is forward. If $\mathbf{n}_p \mathbf{n}_E < 0$, then the direction of movement of phase and energy form an obtuse angle, and the plasmonis is backward. If $\varphi = \psi$ than SP is forward $k'_S k''_S > 0$. If $\psi = \varphi \pm \pi$, then $k'_S k''_S < 0$, and plasmon is backward. In this case σ is scalar, and you can always choose the x axis so that $k_y = 0$. Some results for E-SP are given in Fig. 2 for graphene film using the spatial dispersion model based on BGK approach and without one. Fig. 3 presents the reflection and transmittance for hyperbolic metamaterial (HMM) layer [15, 16] with different angle ϕ between normal and anisotropy axes.

For the problem of diffraction the values k_x and k_y are real and determine the angle φ of inclination of the plane falling to the axis x: $\varphi = \arctan(k_y / k_x)$.

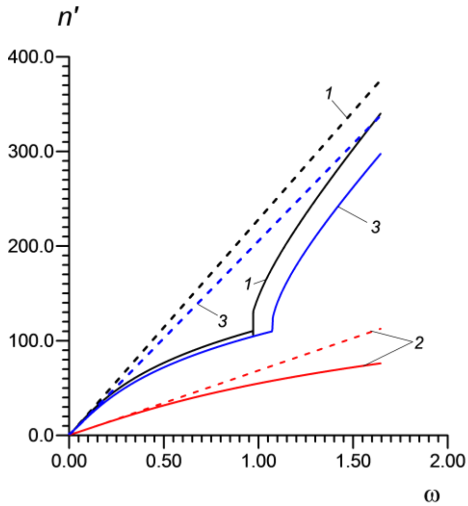

Fig. 2. Deceleration *n'* of E-SP for graphene film at T=300 K, depending on the circular frequency (eV): μ_c=0.3 eV, ε=1 (curves 1); μ_c=1 eV, ε=1 (2); μ_c=1 eV, ε=3 (3) with spatial dispersion (solid curves) and without (dashed curves).

Рис. 2. Замедление n' E-SP для графеновой пленки при T = 300 *K* в зависимости от круговой частоты (эВ): μc = 0,3 эВ, ε = 1 (кривые 1); μ_c = 1 эВ, ε=1 (2); μ_c = 1 эВ, ε = 3 (3) с пространственной дисперсией (сплошные кривые) и без нее (штриховые кривые)

While defining the scattered field value $k_z = \sqrt{k_0^2 - k_x^2 - k_y^2}$ is real, because $k_x^2 + k_y^2 \le k_0^2$. When you go to task about free waves, since non-selfadjoint boundary value problem, the values k_x, k_y, k_z become complex. Such waves should be considered quasieigen and gliding [4, 17], since the leakage from dissipative two-dimensional film is impossible (the structure of the volume and there is no stored energy). The reliance of $\exp\left(-ik_z|z|\right)$ this means that $k_z' < 0$, i. e. the phase and the energy from both parties are moving from a vacuum to the film. This condition is necessary to impose for choice of branches of the root. Leakage is only possible from the active film, for example, from non-equilibrium ("pumped") graphene with σ'<0. Writing $k_z = \sqrt{k_0^2 - k_S^2} = \sqrt{w}$, $w = u + iv = k_0^2 + k_S''^2 - k_S'^2 + 2ik_S' k_S''$, we have $k_z' = -\sqrt{\left(\sqrt{u^2+v^2} + u\right)/2}$, $k_z'' = \pm\sqrt{\left(\sqrt{u^2+v^2} - u\right)/2}$. For forward plasmon the point *w* lies in the upper half plane of complex plane, if so $u = k_0^2 + k_S''^2 - k_S'^2 < 0$ (slow plasmon) the root gives $k_z'' > 0$ (surface plasmon). If

$u = k_0^2 + k_S''^2 - k_S'^2 > 0$ (fast plasmon) it is also surface: $k_z'' > 0$. In active film with forward plasmon (condition $k_S' k_S'' < 0$) the wave is leaky (with $k_z' = \sqrt{\left(\sqrt{u^2 + v^2} + u\right)/2} > 0$) and antisurface ($k_z'' < 0$) since the point *w* lies in the lower half-plane. The case of the backward surface plasmon in dissipative structure $(k_S' k_S'' < 0)$ is characterized by the fact that energy flows into the stricter and phase is leaking. In this case $k_z' > 0$, and you should take the wave surface: $k_z'' > 0$. This case differs from the active one by change of sign k_z. The case of backward plasmon in active structure is characterized in that it emerges at an obtuse angle to the motion of the phase along the surface. In the paper [5] the relations for waves generated by external wave of surface current in the form of a plane are obtained. They imply that the slow wave current creates a surface electromagnetic wave and the fast wave of current – antisurface leaky one, i.e. only quick wave of current radiates. These ratios significantly differ from the above considered: the current is assumed to be specified (incident) and dissipation are not considered. In our case, the problem is self-consistent and with dissipation.

In a thin dielectric film with thickness *t* and the dielectric constant (DC) ε you can enter the current density of the polarization $\mathbf{J}_p = i\omega\varepsilon_0(\varepsilon(\omega) - 1)\mathbf{E}$ and the corresponded surface current density $\mathbf{J}_S = \mathbf{J}_p t$. This suggests that the film has electric conductivity $\sigma^e = i\omega\varepsilon_0 t(\varepsilon(\omega) - 1)$. The result does not depend on the nature of the film. In the case of metal it may be the DC of metal in the form of Drude-Lorentz $\varepsilon(\omega) = \varepsilon_L - \omega_P^2/(\omega^2 - i\omega\omega_c)$. The value $\varepsilon_L - 1$ describes the polarization of the crystal lattice and interband transitions. Similarly, a thin magnetic (e. g., ferrite) films can be described by the magnetic conductivity $\hat{\sigma}^m = i\omega\mu_0 t(\hat{\mu}(\omega) - \hat{I})$ and by surface current of magnetic polarization (magnetization) $\mathbf{J}_S^m = \hat{\sigma}^m \mathbf{H}_S$. For E-wave, we can neglect the transverse component and to calculate the field outside the film based on these formulas and surface current $\mathbf{J}_S = \mathbf{x}_0 t J_{0x} \exp(-ik_x x)$ (consider the motion along the *x*-axis). Then the wave is surface, if $k_x > k_0$ (slow current wave), and it is leaky antisurface [17], if $k_x < k_0$, i. e., during the fast wave of current. These results, however, when dissipation is not quite accurate: you must enter the radiation dissipation $k_x'' > 0$ in accordance with what the objective should be self-consistent. Dissipation shifts the frequency of the transition from slow wave to the fast, which demonstrates the importance of dissipation for the classification of waves.

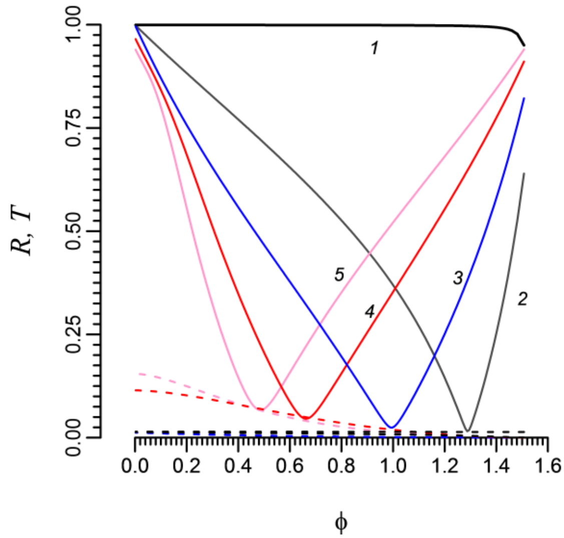

Fig. 3. Dependence of reflection coefficients P (solid curves) and transmission T (dashed curves) of the angle of incidence ϕ for the structure of the HMM with d = 420 nm, $t_m = t_d$ = 20 nm, ε_d = 3 for different values of α: α = 0 (curve 1), α = π/12 (2), α = π/8 (3), α = π/4 (4), α = π/3 (5).

Рис. 3. Зависимость коэффициентов отражения P (сплошные кривые) и пропускания T (штриховые кривые) от угла падения ϕ для структуры ГММ с d=420 нм, $t_m=t_d$=20 нм, ε_d =3 для различных значений α: α = 0 (кривая 1), α = π/12 (2), α=π/8 (3), α=π/4 (4), α=π/3 (5)

For diffraction we choose the *y* axis perpendicular to the plane of incidence. In the incidence of the *p*-polarized wave (E-wave), it has a projection E_{0x}, and in the fall of the *s*-polarized wave it is E_{0y}. In the first case we put $E_{0x}=1$, and then the reflection coefficient is equal to diffraction amplitude $R_p = \overline{E}_{xd}$ or has the form $R_p = T_p - 1 = -1 + a_{22}/\det(\hat{A})$, $R_s = T_s - 1 = -1 + a_{11}/\det(\hat{A})$.

6. Conclusions

We have discusse*d* the symmetric and antisymmetric plasmon-polaritons in symmetric metal-dielectric *and graphebne* structures. In contrast to the customary classification in this work it is carried out by the parity-odd longitudinal component (E_x for E-SP and H_x for H-SP). Accordingly, the even (symmetric) E-SP corresponds to the magnetic wall, and the odd one – to electric *one*. It is shown that the inductive input impedance at the border of the vacuum structure leads to the forward SP, and a capacitive – to the backward SP. That allow one to synthesize the surface metalenses. Dissipation leads to the possibility of propagation of plasmon-polaritons in bandgap zones, and in this case the anomalous negative dispersion does not mean a backward plasmon [6, 7], Fig. 1. The SPs are very slow if the surface input impedance is highly reactive, what for E-SP (3) means: $1-\rho_{in}'^2+\rho_{in}''^2 >> 2\rho_{in}'|\rho_{in}''|$, $\rho_{in}' << |\rho_{in}''|$, $|\rho_{in}''| >> 1$. The impedance approach allows us to obtain DE for both singlet graphene [18] and layered graphene-dielectric structures.

Also we have considered the diffraction of a plane wave on the described by surface conductive films and plane-structures containing such films. The DEs and formulas for the parameters of the scattering have been received. In the general case of anisotropic conductivity the modes are hybrid.

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References

1. Tournoisa P., Laude V., Negative group velocities in metal-film optical waveguides // Optics Communications. 1997. Vol. 137. P. 41–45.
2. Negative group velocity of surface plasmons on thin metallic films / Y. M. Liu [et al.] // Proc. SPIE. 2006. Vol. 6323, P. 63231M–8.
3. Surface plasmon-polaritons with negative and zero group velocities propagating in thin metal films / D. Yu. Fedyanin [et al.] // Quantum Electron. 2009. Vol. 39, No. 8. P.745–750.
4. Zuev V. S., Zueva G. Ya. Very slow surface plasmons : Theory and practice (Review) // Optics and Spectroscopy. 2008. Vol. 105. P. 852–859.
5. Davidovich M. V. Plasmons in multilayered plane-stratified structures // Quantum Electron. 2017. Vol. 47, No. 6. P. 567–579.
6. Davidovich M. V. Maximum deceleration and negative dispersion of plasmons along a metal layer // Technical Physic Letters. 2017. Vol. 43, No. 22. P. 55–62.
7. Electrodynamics of carbon nanotubes : Dynamic conductivity, impedance boundary conditions, and surface wave propagation / G. Ya. Slepyan [et al.] // Phys. Rev. 1999. Vol. B 60, No. 24. P. 17136–17149.
8. Hanson G. W. Dyadic Greens functions and guided surface waves for a surface conductivity model of graphene // J. Applied Phys. 2008. Vol. 103. P. 064302(1–8).
9. Maksimenko S. A., Slepyan G. Ya. Electrodynamic properties of carbon nanotubes. In : Electromagnetic Fields in Unconventional Structures and Materials, Ed. by O. N. Singh and A. Lakhtakia. New York : John Wiley & Sons, Inc., 2000. P. 217—255.
10. Falkovsky L. A. Dynamic properties of graphene // JETP. 2012. Vol. 115, No. 3. P. 496–508.
11. Semiclassical spatially dispersive intraband conductivity tensor and quantum capacitance of graphene / G. Lovat [et al.] // Phys. Rev. 2013. Vol. B 87, No. 11. P. 115429(1–11).
12. Brazhe R. A., Kochaev A. I., Miftahutdinov R. M. Graphens and their physical properties : a tutorial. Ulyanovsk, Russia : UlSTU Press, 2016. 139 p. [In Russ.].

13. Markov G. T., Chaplin A. F. Excitation of electromagnetic waves. Moscow, Russia : Radio i Sviyaz, 1983. 296 p. [In Russ.].
14. Rytov S. M. Certain theorems on the group velocity of electromagnetic waves // Zh. Eksper. Teoret. Fiz. 1947. Vol. 17, No. 10. P. 930–936.
15. Davidovich M.V. Polaritons and diffraction on the layer of asymmetric hyperbolic metamaterial // Proc. of SPIE. 2020. Vol. 11458. P. 1145813(1–7).
16. Davidovich M. V. Dyakonov plasmon-polaritones along a hyperbolic metamaterial surface // Computer Optics. 2021. Vol. 45, No. 1. P. 48–57.
17. Vainshtein L. A. Electromagnetic waves. Moscow, Russia : Radio i Svyaz, 1988. 440 p. [In Russ.].
18. Mikhailov S. A., Ziegler K. New electromagnetic mode in graphene // Phys. Rev. Lett. 2007. Vol. 99. P. 016803(1–4).

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